Non-Abelian states of matter

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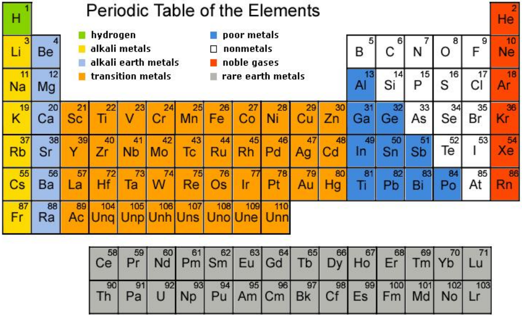
Fundamental particle statistics

Dichotomy

- All particles are either bosons or fermions
- Distinguishable by the symmetry of the wave function

Key element for understanding many-particle systems (e.g. periodic table, metals, cooper-pairs, BEC...)



Violation of the dichotomy

- The wave function must be even for bosons and odd for fermions under the particle exchange $\vec{r}_i, \vec{r}_j \rightarrow \vec{r}_j, \vec{r}_i$
- **Non-abelian systems contain particles that are neither bosons nor fermions**, i.e. the wave functions don't get multiplied by the factor ± 1 but the system changes its state into a different state!
- For a series of particle interchanges **the final state of the system depends on the order** of these interchanges!
- Due to this ordering-dependence such systems are called *non-abelian* (non-commutative)

Interest in non-abelian systems

- There are three main reasons for studying non-abelian systems:

1. Theory of such systems is barely known
2. Experimental challenge: there are strong arguments for the existence of non-abelian particles in certain systems
3. They could be used in topological quantum computers

Non-abelian states

In the past 3 decades quasiparticles in 2 dimensions have been discovered whose particle statistics differ from the fermion-boson-dichotomy

- Under a particle interchange the wave function may get multiplied by a phase factor $\psi(\vec{r}_i, \vec{r}_j) \rightarrow e^{i\phi}\psi(\vec{r}_j, \vec{r}_i)$
- These quasiparticles are called *anyons*
- For *non-abelian* quasiparticles the interchange of two particles shifts the system from its groundstate into another $\psi_\alpha(\vec{r}_i, \vec{r}_j) \rightarrow \psi_\beta(\vec{r}_j, \vec{r}_i)$

Non-abelian states

There are 4 main characteristics or prerequisits for non-abelian states to occur:

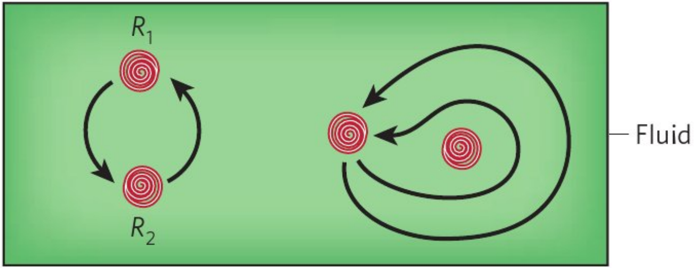
1. The groundstate of the system has to be seperated from its excitations by an **energy gap**
2. The groundstate must be **degenerate**
3. The degeneracy should be '**robust**', i.e. invariant to small perturbations
4. The **transformation** to a different groundstate should **only depend on the topology** of the interchanging particles' trajectory

2D-fluid

To understand what topology of the trajectory means let us consider a 2-dimensional fluid with some vortices which shall represent our non-abelian quasiparticles:

- Let the groundstate of the system depend on the coordinates of the vortices $\{\vec{R}_i\}$
- With a degenerate ground state we have a set of groundstate wave functions labeled by the Index α: $|\psi_\alpha(\{\vec{R}_i\})\rangle$
- '**Braiding**': the quasiparticles move slowly along a trajectory that starts and ends in the same configuration $\{\vec{R}_i\}$

Non-degenerate groundstate



- For a **single non-degenerate groundstate** the system will stay in this groundstate (up to a phase factor) for gentle braiding due to the energy gap: $|\psi_\alpha(\vec{R}_i)\rangle \rightarrow e^{i\phi}\,|\psi_\alpha(\vec{R}_i)\rangle$
- Consecutive braidings result in a **product of phase factors** which is commutative

$\rightarrow$ **Quasiparticles are abelian anyons!**

Degenerate groundstate

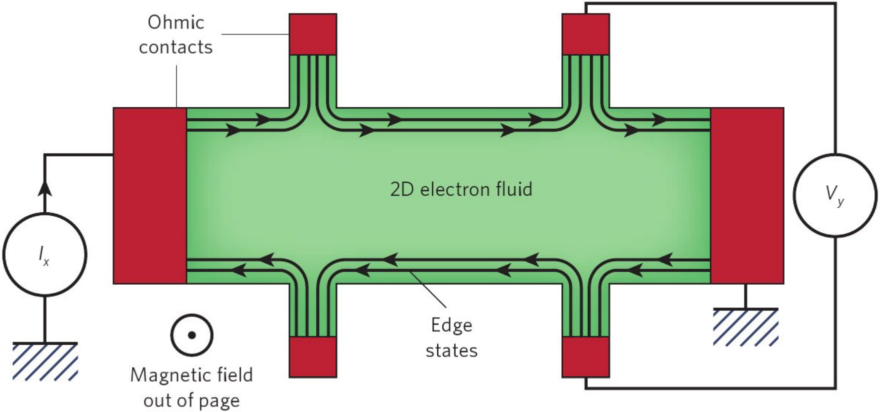
- For a **degenerate groundstate** braiding could end up with a different groundstate $|\psi_\alpha(\vec{R}_i)\rangle \rightarrow |\psi_\beta(\vec{R}_i)\rangle$
- Such transformation is no longer given by a phase factor but a unitary matrix $U_{\alpha\beta}$:
  $|\psi_\beta(\vec{R}_i)\rangle = U_{\alpha\beta}\,|\psi_\alpha(\vec{R}_i)\rangle$
- $U_{\beta\alpha}$ does only depend on the topology of the trajectory, i.e. the interchanges and windings around one another, and not on the geometric details!
- Consecutive braidings result in a **product of unitary matrices**, which is itself non-commutative

$\rightarrow$ **Quasiparticles are non-abelian anyons!**

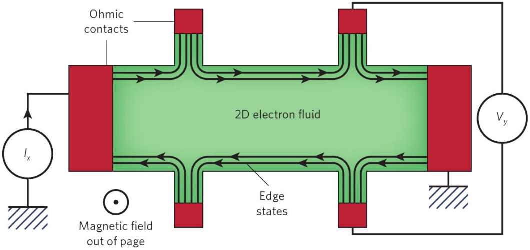
- Thus a high degree of degeneracy is needed
- A 'robust' degeneracy of the groundstate is necessary for the emergence of non-abelian particles
- The groundstate has to be seperated by an energy gap

- For certain systems where these criteria are met, like the quantum Hall effect, there is strong evidence for non-abelian states

Quantum Hall Effect



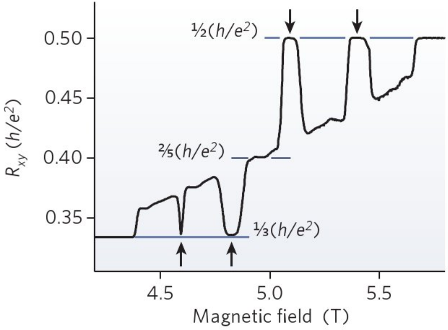
Quantum Hall Effect



- 2D electronic system with is subjected to an external field
- A current flow $\vec{j} \perp \vec{B}$ gives rise to the Hall-Voltage U_H
- Hall resistance: $R_H = \frac{U_H}{I} = \frac{h}{e^2 \nu}$
- Classical filling factor $\nu = \frac{nhc}{eB}$

Quantum Hall Effect

- For small T one observes quantum hall states
- **Quantized** ν leading to plateaus in R_H
- **Integer** QHE: $\nu = \{1, 2, 3, ...\}$ non-interacting electrons with disordered potential
- **Fractional** QHE: $\nu = \{1/3, 1/5, 5/2, 12/5, ..\}$ interaction essential



Composite fermions

- The groundstate for $\nu = 5/2$ is a hot candidate for a non-abelian state because it is high degenerate and seperated by an energy gap

A promising theory for this state is the **composite fermion theory**:

- The problem of interacting electrons in a field B is mapped onto a problem of composite fermions with a rescaled B
- These composite fermions may form **Cooper pairs** that are seperated from their excitations by an **energy gap**
- Then quasiparticles can be **induced** by variation of the magnetic field which are supposed to be non-abelian

Superconductors

- If these quasiparticles are indeed non-abelian such a system could be used as a topological qubit (by interferometry)

- Non-abelian states are also expected in superconducters (e.g. $SrRuO_4$)
- The quasiparticles may obey non-abelian statistics
- None of these superconducters has been realized in 2 dimensions yet

Detection

The degeneracy of the groundstate poses an experimental difficulty because the usual approach of perturbing the system and examining its response **doesn't work**:

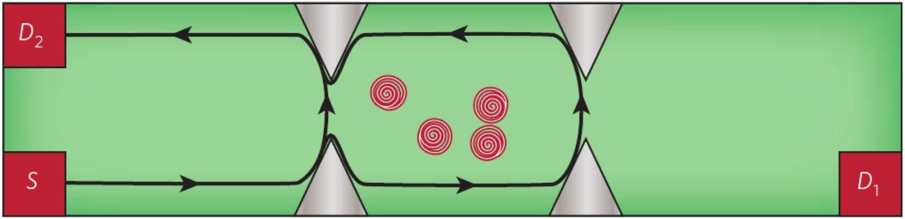
- Perturbations such as electric fields **do not couple** to the different groundstates **equally**
- Nevertheless, the $\nu = 5/2$ quantum Hall state is currently of great interest and several experiments have been proposed

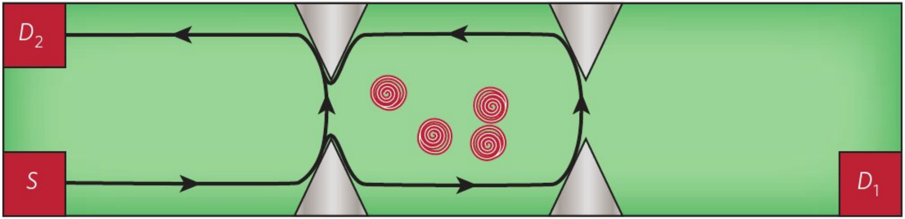
$\nu = 5/2$ quantum Hall state

- Being a key thermodynamic ingredient the degeneracy should manifest itself in thermodynamic measurements such as the **heat capacity**
- But the **small electronic contribution** to C_V is neglegible compared to phonons, etc.
- One can measure the temperature dependence of the **electronic magnetization** and **chemical potential** which are connected to the electronic entropy (Maxwell relations)
- This measurements are already possible for 2D systems but have not been carried out for the $\nu = 5/2$ state

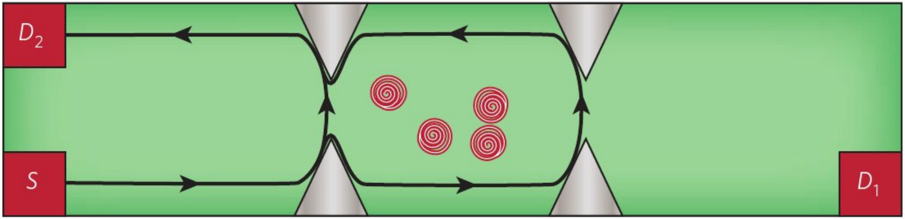
Interferometry

- One of the most important aspect is to examine and make use of the groundstate transformation by braiding quasiparticles
- **Interferometry experiments** with **quantum Hall systems** could be ideal for detecting the transformations and might be used as qubits:

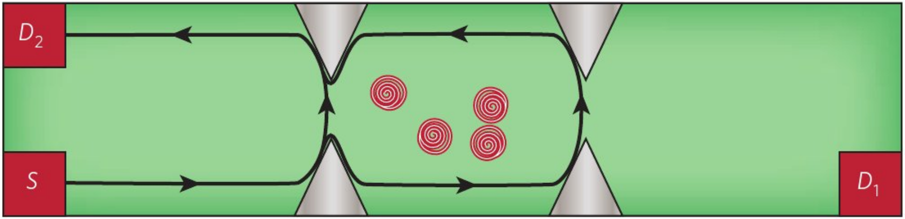




- An interferometer interferes pairs of wave trajectories which gives rise to an **interference pattern**
- The **Fabry-Perot** interferometer is a quantum Hall system with two **constrictions** that introduce a **tunneling probability** for incoming current
- So we have interference between the **backscattered** and **transmitted** current



- The **phase difference** is given by the magnetic flux enclosed by the interference loop (Aharonov-Bohm effect) and also by the presence of **quasiparticles** inside the loop
- For the $\nu = 5/2$ state interference takes place for an even number of quasiparticles depending on their specific groundstate
- The number of quasiparticles can again be controlled with the strength of the external magnetic field



- So the interference should be turned on and off periodically while varying the magnetic field
- The pattern itself **depends on the groundstate** of the quasiparticles and one should be able to change it by braiding these particles
- This could be used as the proposed qubit where a measurement of the interference pattern serves as a read-out of its state
- The state itself is controlled by external fields

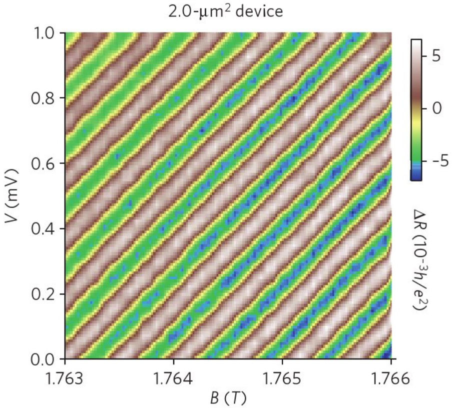
- This qubit is called **topological qubit** due to the non-abelian behaviour of the quasiparticles (groundstate depends on topology of trajectories)
- The advantage of this kind of qubit is its **insensivity to perturbations** and **external noise**
- As long as the perturbation does not overcome the energy gap or expels one of the quasiparticles from the interference loop it will not change the state

Current status

- There is a huge difference between the discussed ideal picture and the real world
- One can not or even does not know how to control the number and positions of the quasiparticles
- Effects of the temperature, small energy gaps or tunnel couplings to the edge of the material still need to be understood

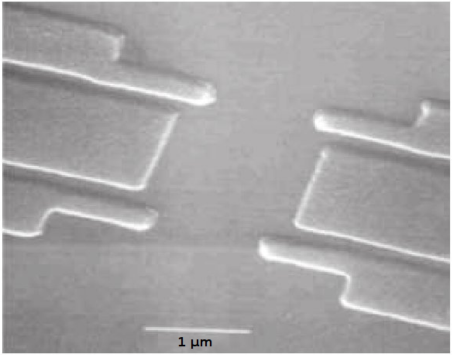
Quantum Hall states

- Fabry-Perot interferometers have been studied for $\nu = 1/3$ and $2/5$ and oscillations depending on the external field have been observed
- But the effects on the size of the interference loop, the number and the groundstate of the quasiparticles seem to be different than expected



The $\nu = 5/2$ state

- A Fabry-Perot interferometer for the $\nu = 5/2$ state has been realized and recent studies yield 'encouraging, yet not definitive results'.



Outlook

- The current experiments are aimed at the observation of non-abelian states and much has to be learned about probing states which are insensitive to perturbations
- Non-abelian statistics which are topology dependent pose an exciting challenge to theoretical physics
- Knowledge about these states could be used for the synthesis of new materials, electronic systems, cold atoms, quantum computation...
- One hopes that in the next decade the combination of quantum mechanics, topology and material science will make non-abelian states a reality