Negative Refractive Index and Metamaterials

Jean Diehl

Institut für Theoretische Physik
Goethe-Universität, Frankfurt

July 6th, 2011

Contents

Maxwell's Equations

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon} \qquad \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0 \qquad \nabla \times \mathbf{B} = \mu \mathbf{j} + \epsilon\mu \frac{\partial \mathbf{E}}{\partial t}$$

- no conductivity: wave equations

$$\nabla^2 \mathbf{E} = \epsilon\mu \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

- assume linear response $\mathbf{j} = \sigma \mathbf{E}$: Telegrapher's equations

$$\nabla^2 \mathbf{E} = \mu\sigma \frac{\partial \mathbf{E}}{\partial t} + \epsilon\mu \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

- Solution: plane waves $\mathbf{E} = \mathbf{E_0} e^{i(\mathbf{k}\cdot\mathbf{r} - \omega t)}$ with dispersion relation

$$k^2 = \frac{\omega^2}{c^2} + i\omega\mu\sigma \qquad \text{mit } c^2 \equiv \frac{1}{\epsilon\mu}$$

Refractive Index

- introduce complex permittivity to use wave equation again:

$$\epsilon(\omega) = \epsilon' + i\epsilon'' = \epsilon + i\frac{\sigma}{\omega}$$

- analogously complex value for permeability:

$$\mu(\omega) = \mu' + i\mu''$$

- definition of the refractive index:

$$n \equiv \frac{c_0}{c} = \sqrt{\epsilon_r \mu_r}$$

- assume $\epsilon = \mu = -1 = e^{i\pi}$:

$$n = \sqrt{e^{i2\pi}} = e^{i\pi} = -1$$

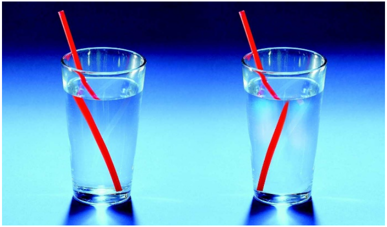
Negative Refractive Index

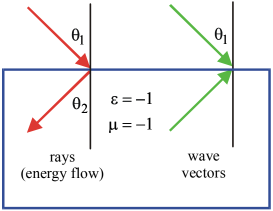
- first theoretical description: V. G. Veslago[1]
- called: left handed materials

$$\mathbf{k} \times \mathbf{E} = \frac{\omega\mu}{c}\mathbf{H} \qquad \mathbf{k} \times \mathbf{H} = -\frac{\omega\epsilon}{c}\mathbf{E}$$

- Snells law has to be modified:

$$\frac{\sin\alpha}{\sin\beta} = \frac{p_2}{p_1}\sqrt{\frac{\epsilon_2\mu_2}{\epsilon_1\mu_2}}$$

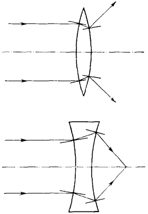


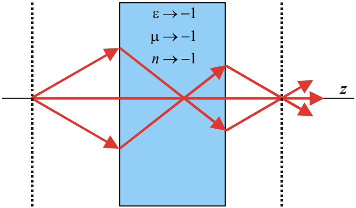


[1]V. G. Veselago, *Sov. Phys. Usp.* **10**, 509 (1968)

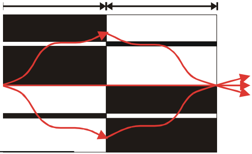
Phenomena and Resulting Applications

- Doppelereffect is inversed
- Cherenkov-Radiation is inversed
- classical lenses act inversed





- superlenses[2] with better resolution
- optical antimatter



[2] J.B. Pendry, *Phys. Rev. Lett.* **85** 3966 (2000)

Phenomena and Resulting Applications

- cloaking device to hide

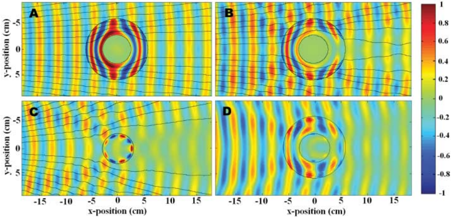
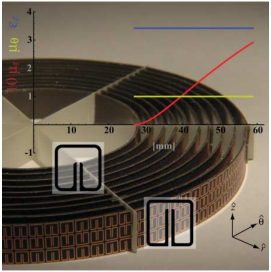


Fig. 4. Snapshots of time-dependent, steady-state electric field patterns, with stream lines [black lines in (A to C)] indicating the direction of power flow (i.e., the Poynting vector). The cloak lies in the annular region between the black circles and surrounds a conducting Cu cylinder at the inner radius. The fields shown are (**A**) the simulation of the cloak with the exact material properties, (**B**) the simulation of the cloak with the reduced material properties, (**C**) the experimental measurement of the bare conducting cylinder, and (**D**) the experimental measurement of the cloaked conducting cylinder. Animations of the simulations and the measurements (movies S1 to S5) show details of the field propagation characteristics within the cloak that cannot be inferred from these static frames. The right-hand scale indicates the instantaneous value of the field.



[3]

[3] D. Schurig et al. *Science* **314**, 977-980 (2006)

Metamaterials

- periodic structure is smaller than wavelength
- microwave region (swiss rolls, split-ring-resonators)
- most recently optical (cut wires, fishnet)

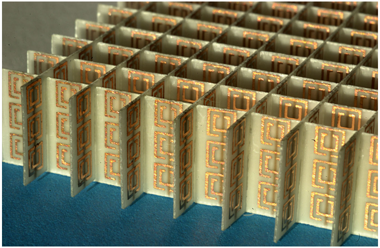
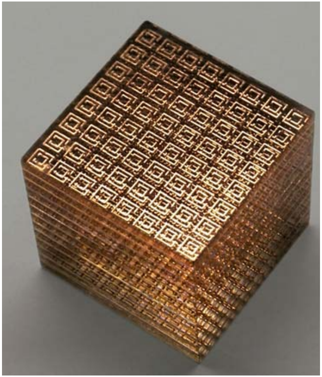
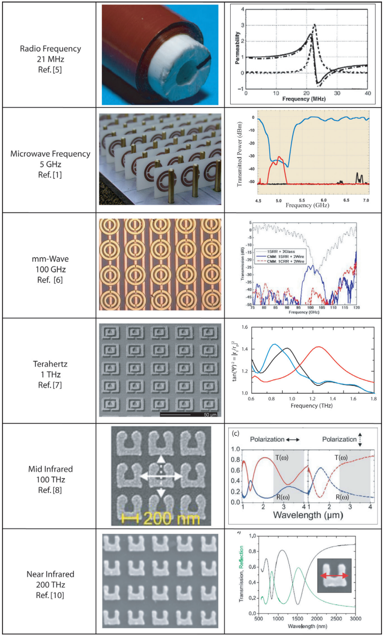


Figure 11. A split ring structure etched into copper circuit board plus copper wires to give negative μ and negative ε. Structure made at UCSD by David Smith.



'The Boeing cube': a structure designed for negative refractive index in the GHz range

Overview



[4]

[4] W. J. Padilla, D. N. Basov, D. R. Smith *Mat. Today.* **9**, 28-35 (2006)

Infrared Region

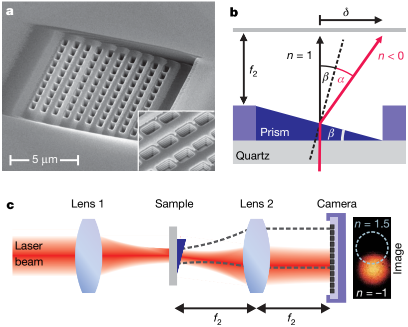


Figure 2 | SEM image of NIM prism and schematics of experimental setup. **a**, SEM image of the fabricated 3D fishnet NIM prism. The unit cell size is identical to that shown in Fig. 1a. The inset shows a magnified view with the film layers visible in each hole. **b**, Geometry diagram of the angle measurement; δ corresponds to the position difference of the beam passing through a window in the multilayer structure ($n = 1$) and prism sample. By measuring δ, the absolute angle of refraction α can be obtained. [5]

[5] J. Valentine et al. *Nature* **455**, 376–379 (2008)

Infrared Region

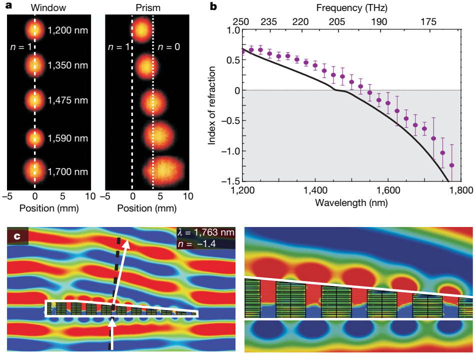


Figure 3 | Experimental results and finite-difference time-domain simulations. a, Fourier-plane images of the beam for the window and prism sample for various wavelengths. The horizontal axis corresponds to the beam shift δ, and positions of $n = 1$ and $n = 0$ are denoted by the white lines. The image intensity for each wavelength has been normalized for clarity. **b**, Measurements and simulation of the fishnet refractive index. The circles show the results of the experimental measurement with error bars (s.d., $n = 4$ measurements). The measurement agrees closely with the simulated refractive index using the RCWA method (black line). **c**, Left: simulation of the in-plane electric field component for the prism structure at 1,763 nm, showing the phase front of the light. Negative-phase propagation resulting from the negative refractive index leads to negative refraction angle as measured by the beam shift in the experiment. Right: magnified plot of the field distribution in the prism. [6]

[6] Valentine, J. et al. *Nature* **455**, 376–379 (2008)

Outlook

- negative refractive index over wide wavelength range
- negative permeability already achieved over complete optical region[7]
- dissipation is a problem
- processing of nanometer scale structures

[7] C. Wenshan et al. *Optics Express* **15**, 6 p.3333 (2007)